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FUNCTIONAL OF RECEPTOR SENSITIVITY AND ADJOINT EQUATIONS

V. P. Reshetin[a] **and D. Regens** [b]

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The functional of receptor sensitivity of a territory to the disposition of emission sources can adequately be calculated with the use of the mathematical apparatus of adjoint equations. The solutions of these equations allow one to determine the action of pollutants in the form of aerosols or harmful gaseous impurities on the environment and human health. Examples of calculation of the sensitivity functional on the basis of solution of adjoint equations have been presented. Mathematical models for the typical equations on optimum placement of industrial plants have been formulated. An interpretation of the results obtained has been given.

Recent investigations point to the fact that atmospheric pollution leads to a rise in the sickness rate and premature-death rate [1–6]. For example, unique epidemiological investigations [3] have revealed a coherent and statistically reliable relation between the pollution of the atmosphere with small suspended particles of size less than 10 μm and the death rate. Evaluations of the number of attributed deaths in France, Austria, and Switzerland have shown that about 6% of all the lethal outcomes in these countries are due to pollution of the atmosphere with small particles of size less than 10 μm [4]. Taking into account the fact that in Russia the level of exposure to pollutants is higher, it may be suggested that the number of attributed deaths caused by atmospheric pollution can be much higher and can reach 16–17% of the number of all lethal outcomes [17].

Because of this, the problem of decreasing the hazard of atmospheric pollution to human health becomes pressing. One way of solving it is optimization of the location of new industrial plants and complexes. A less important problem is optimization of a decrease in the level of ejection of harmful chemical substances at already existing industrial plants. Since labor resources are distributed irregularly, such plants are usually constructed in densely populated regions or in the immediate vicinity of them. As for the health of people and the environment, the positive effect of a decrease in the level of emissions substantially depends on the location of the emission sources.

The problem of optimum placement of new industrial plants and efficient decrease in the level of emissions at already existing plants can be solved on the basis of calculation of the receptor-sensitivity functional [1]. With this functional, one can quantitatively evaluate the change in the hazard to human health depending on the location of an emission source. The mathematical apparatus of adjoint equations, developed by G. I. Marchuk [8], provides a new means for calculation of receptor sensitivity. In the present work, the problem of optimum placement of industrial plants is investigated with the use of the data obtained in [1, 8], and the mathematical models of the most typical situations are considered.

Formulation of the Problem. It will be assumed that a new industrial plant must be positioned near populated areas or directly on the territory of a large settlement, provided that the hazard of atmospheric pollution to the health of people living in this region Σ_0 is minimum or within certain permissible norms at each point of the region. Let an aerosol source $f(\mathbf{r})$ be positioned at the point $\mathbf{r}_0 = (x_0, y_0, z_0)$ and have intensity Q:

$$f(\mathbf{r}) = Q\delta(\mathbf{r} - \mathbf{r}_0) . \tag{1}$$

By the action of wind, the impurity is carried by air masses and diffuses under the influence of small-scale convection. In the simplest formulation, this process in the atmosphere can be described by the equation

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[a] Joint Institute for Energy and Nuclear Research, National Academy of Sciences of Belarus, Minsk, Belarus; email: reshetin@sosny.bas-net.by; [b] Institute for Science and Public Policy, University of Oklahoma, Oklahoma, USA. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 76, No. 4, pp. 53–60, July–August, 2003. Original article submitted December 19, 2002.

$$\frac{\partial C}{\partial t} + \operatorname{div}(\mathbf{U}C) + \lambda C - K_x \frac{\partial^2 C}{\partial x^2} - K_y \frac{\partial^2 C}{\partial y^2} - \frac{\partial}{\partial z} K_z \frac{\partial C}{\partial z} = Q\delta(\mathbf{r} - \mathbf{r}_0) . \tag{2}$$

The problem will be solved in a cylindrical region with boundary conditions

$$C\big|_{\Sigma} = 0 , \quad \left.\frac{\partial C}{\partial z}\right|_{\Sigma_0} = \alpha C , \quad \left.\frac{\partial C}{\partial z}\right|_{\Sigma_H} = 0 . \tag{3}$$

We will also assume that the function C is periodic with period T:

$$C(\mathbf{r}, T) = C(\mathbf{r}, 0) . \tag{4}$$

To evaluate the hazard to the health of people living in the region Σ_0, we multiply the concentration of the aerosol near the earth's surface by the density of the population $P(r)$ and integrate the function obtained over the area of the region (over the period of time T):

$$F = a \int_0^T dt \int_{\Sigma_0} PC d\Sigma . \tag{5}$$

Here $a = b/T$, the constant b reflecting the dose–response relation, whose numerical values have been determined, e.g., in [3]. The functional (5) determines within a multiplier the collective exposure (dose), averaged over the period of time T, to which the population of the region will be subjected as a result of the emission of aerosols by the source (1). At a given location of an aerosol source, the functional (5) estimates the action of this source on the population. If this estimation is carried out with the use of the dose–response function from [3], which determines the relation between exposure and premature-death rate, functional (5) will estimate the number of lethal outcomes caused by atmospheric pollution. Having calculated the functional (5) for emission sources positioned at different points of the region, one can determine how atmospheric pollution influences the death rate depending on the location of the source. In [1], the functional (5) is called the receptor sensitivity of the territory to the location of emission sources.

It should be noted that at the existing level of pollution, the relation between the exposure and the response is linear for many effects harmful to the health of people and the environment. The distribution of the functional over the territory of a region estimates the sensitivity of the territory to the location of an emission source [1]. If the intensity and the location of the source are known, the value of the functional (5) depends on the wind rose characteristic of this region, the lay (relief) of the ground, and the features of the population distribution over the territory of the region. Because of the linearity of the dose–response function, the effect of the action of several sources on the population and the environment is an additive quantity. Thus, the receptor sensitivity of the territory is independent of the location of the already existing emission sources in the region. Among other things, the distribution of the receptor sensitivity over the region allows one to analyze the efficiency of a decrease in the emission of one industrial plant or another from the viewpoint of decreasing hazard; in this case, at the same decrease in emission, the decrease in the hazard will be the largest for the enterprises positioned on the territory with a high value of the sensitivity functional [1].

A functional analogous to the functional (5) can be used for evaluating the environmental hazard. However, depending on the chosen priorities, the distribution of some parameters or others significant for its evaluation should be used for the function $P(r)$. Since the environmental hazard is usually evaluated taking into account the effects on the population of a community or an ecological system, their dynamics and structure along with the processes in the ecological system are those endpoints on which the evaluation of the hazard is usually concentrated. Since there are no universal environmental endpoints, the evaluation of the hazard and subsequent calculation of the receptor sensitivity of the territory should be limited most probably to a concrete situation. When the function $P(r)$ is determined, the resources potentially exposed to the emission products should be considered first. In the process of identification of the endpoints of evaluation of the hazard and determination of the function $P(r)$, it is necessary to analyze conceptional models, environmental effects, and other factors.

Since the main functional of the problem has been selected in the form of (5), the adjoint equations will be as follows:

$$-\frac{\partial C^*}{\partial t} - \operatorname{div}\,(\mathbf{U}C^*) + \lambda C^* - K_x \frac{\partial^2 C^*}{\partial x^2} - K_y \frac{\partial^2 C^*}{\partial y^2} - \frac{\partial}{\partial z} K_z \frac{\partial C^*}{\partial z} = P\,(r)\,\delta\,(z)\,,$$

$$C^* \big|_{\Sigma} = 0\,, \quad \left.\frac{\partial C^*}{\partial z}\right|_{\Sigma_0} = \alpha C^*\,, \quad \left.\frac{\partial C^*}{\partial z}\right|_{\Sigma_H} = 0\,, \quad C^*\,(\mathbf{r}, T) = C^*\,(\mathbf{r}, 0)\,. \tag{6}$$

Because of the adjointness of Eqs. (6), the functional (5) can be written in the form (dual representation of the functional [5])

$$F = aQ \int_0^T C^*\,(\mathbf{r}_0, t)\,dt\,. \tag{7}$$

The functional (7) is parametrically dependent on the site of disposition of an aerosol source. At $z = 0$, the solution of the associated problem determines the time dependence of the collective exposure of the population of the region C^* on the location of an emission source of unit intensity. Thus, the appealing aspect of the adjoint equations becomes evident: their solution makes it possible to determine the collective exposure; to calculate the receptor sensitivity of the territory of the region it is necessary to simply average this exposure over any period of time and multiply it by the coefficient determined by the dose–effect dependence. Unlike the main problem, in which to calculate the functional (5) it is necessary to find the distribution of the aerosol concentration for each site of the emission source, the adjoint equations make it possible to calculate it using just one calculation variant.

In some cases, the functional (5), which is equal within a multiplier to the exposure averaged over the period T, can be calculated as a superposition of the stationary solutions of the adjoint equations (6):

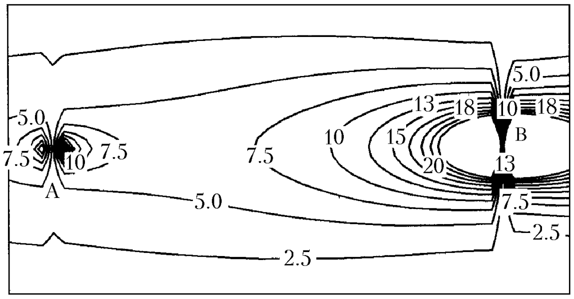
$$F = aQ \sum_{i=1}^{n} C_i^* \Delta t_i\,, \quad \text{where} \quad \sum_{i=1}^{n} \Delta t_i = T\,. \tag{8}$$

The stationary solutions of the adjoint equations can be used for averaging the exposure C^* over the wind directions with account for the wind rose in the region. The averaging over the stationary solutions makes it possible to calculate to sufficient accuracy the receptor sensitivity in the case where the contribution of the transient processes is insignificant.

Mathematical Models of Typical Situations. As the simplest example, we consider the following problem. Let us assume that an industrial plant ejecting harmful aerosols into the atmosphere must be positioned between two settlements A and B with the number of people P_1 and P_2, found at the points with coordinates (0, 0, 0) and (L, 0, 0). We will assume for simplicity that the wind blows with a velocity U_1 in the direction of the settlement A during the period of time Δt_1 and with a velocity of U_2 in the direction of settlement B during the period of time Δt_2 and $T = \Delta t_1 + \Delta t_2$. Let us determine at what distance from settlements the plant must be positioned in order that the hazard to human health be minimum. In this case, the stationary solution of the adjoint equations has the following form:

$$C^* = \frac{P_1 \Theta\,(x_0)}{4\pi\,(K_y K_z)^{1/2} r_1} \exp\left[-\frac{U_1}{4r_1}\left(\frac{y_0^2}{K_y} - \frac{z_0^2}{K_z}\right)\right] + \frac{P_2 \Theta\,(L - x_0)}{4\pi\,(K_y K_z)^{1/2} r_2} \exp\left[-\frac{U_2}{4r_2}\left(\frac{y_0^2}{K_y} - \frac{z_0^2}{K_z}\right)\right], \tag{9}$$

where $r_1 = (x_0^2 + y_0^2 + z_0^2)^{1/2}$ and $r_2 = ((L - x_0)^2 + y_0^2 + z_0^2)^{1/2}$. The distribution of the receptor-sensitivity functional along the straight line connecting the settlements is described by the expression



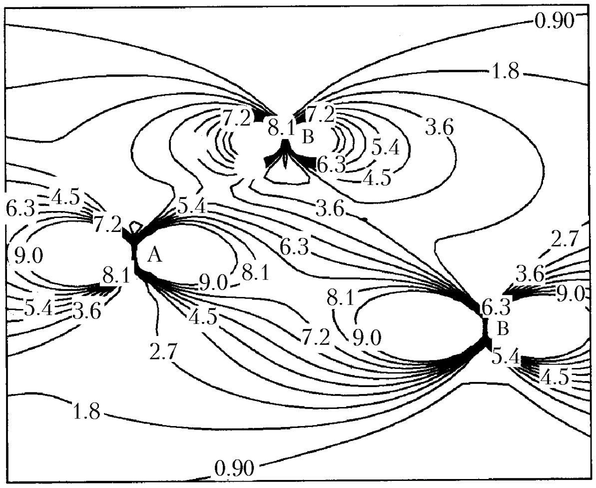


Fig. 1. Distribution of the receptor-sensitivity functional (rel. units) in the region between two settlements. The computational region measures 1 × 1 km. $P_A:P_B = 10:1$, $K_y = 10^2\ \mathrm{m^2/sec}$, $K_z = 10\ \mathrm{m^2/sec}$, $U_1 = 3$ m/sec, $U_2 = 5$ m/sec, and $\Delta t_1/\Delta t_2 = 2/3$.

Fig. 2. Distribution of the receptor-sensitivity functional (rel. units) in the region with three settlements. The functional was calculated by the semiempirical ASME model for the neutral atmospheric state [11]. The computational region measures 1 × 1 km. $P_A:P_B:P_C = 8:5:10$, $U_1 = 3$ m/sec, $U_2 = 5$ m/sec, $\Delta t_1/\Delta t_2 = 2/3$, and $h = 0$.

$$F(x_0, 0) = \frac{aQP_1\Theta(x_0)}{4\pi (K_yK_z)^{1/2} x_0}\,\frac{\Delta t_1}{T} + \frac{aQP_2\Theta(L - x_0)}{4\pi (K_yK_z)^{1/2}(L - x_0)}\,\frac{\Delta t_2}{T}. \tag{10}$$

Because of the linearity of the problem, F represents, within a multiplier, a superposition of the collective exposures for each settlement. The collective exposure is inversely proportional to the distance from the emission source. The terms in (10) can be interpreted as the hazard potentials of the settlements, and the receptor sensitivity of the territory represents the sum of the hazard potentials for all the settlements. The minimum value of the functional F is attained in the case where the ratio between the distances from the emission source to the settlements is equal to

$$\frac{x_1}{x_2} = \left(\frac{P_1\Delta t_1}{P_2\Delta t_2}\right)^{1/2}. \tag{11}$$

For example, at $\Delta t_1 = \Delta t_2$ and the ratio between the number of people in the settlements $P_1/P_2 = 9$, the minimum hazard to human health will be in the case where the emission source is found at a distance of $L/4$ from settlement A. The distribution of the receptor-sensitivity functional for this example is presented in Fig. 1. If one more settlement, C, appears in the region studied, the receptor-sensitivity distribution reflects the contribution of three hazard potentials (see Fig. 2) now, each of which can be calculated in the same way as in the problem with two settlements.

In the case of an arbitrary distribution of the wind velocity in directions, to calculate the sensitivity potential it is necessary to sum the collective exposures in (7) for all the directions of the wind with account for the data on the wind rose in the region considered. In the case where the wind is directed at an angle φ to the abscissa axis, the following transformation of coordinates makes it possible to pass to the coordinate system (x', y', z'), where the wind velocity is directed along the abscissa axis, as before, and so the above-indicated expressions for the hazard potential (10) can be used in calculating the receptor sensitivity:

$$x = x'\cos\varphi - y'\sin\varphi, \quad y = x'\sin\varphi + y'\cos\varphi.$$

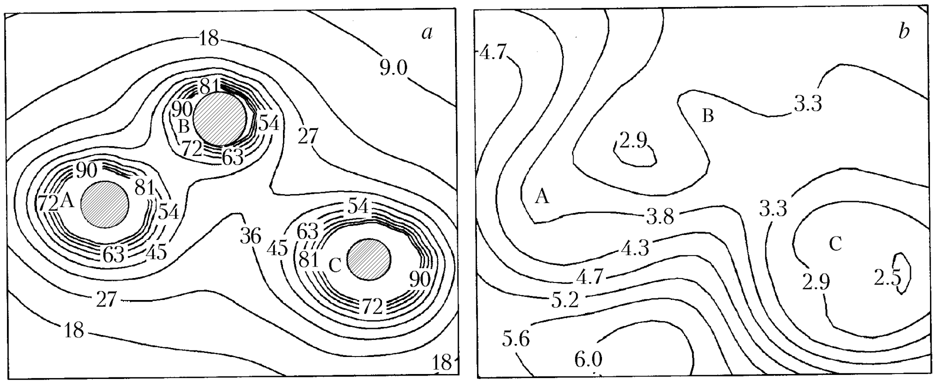


Fig. 3. Distribution of the receptor-sensitivity functional (rel. units) in the region with three settlements: a) $h = 0$ and b) 50 m. The functional was calculated by the semiempirical ASME model for the stable atmospheric state [11]. The computational region measures 1 × 1 km. $P_A:P_B:P_C = 8:5:10$. The data on the wind rose characteristic of Minsk (Table 1) were used in the calculations.

TABLE 1. Average Annual Wind Rose in Minsk

Wind direction	North	Northeast	East	Southeast	South	Southwest	West	Northwest	Calm
Recurrence, %	8	9	9	13	15	14	17	15	5
Mean velocity, m/sec	3.1	3.1	3.1	3.0	2.9	3.0	3.3	3.4	0

It should be noted that in the present work we used the simplest model of aerosol transport in the atmosphere, which does not take into account the dependence of the diffusion coefficients on the height and the distance from the emission source and the effect of rise of the cloud due to the initial angular momentum and the action of the buoyancy force. The known computer codes describing impurity transport in the atmosphere can be used for investigating the influence of these effects. In particular, such an investigation can be carried out with the use of the "Nostradamus" code [9] based on modern understanding of the structure of the boundary layer, which makes it possible to describe impurity transport in a given wind field with the use of the data of regional weather stations. Because of the similarity of the problems (2), (3), and (6), it is easy to set up computer programs for solution of the adjoint equations. In the present work, for simplicity we assumed that the settlements are point objects; this assumption can also be suitable in a number of cases for area objects (the problem can be easily generalized). In the case where a settlement cannot be considered as a point object, functionals (7) and (8) represent the result of integration over all territories of the region where people live.

In almost all countries of the world, the maximum permissible concentrations of harmful chemical substances are established by law. Therefore, the problem of minimization of the hazard to human health must be solved so that the concentrations of aerosols in settlements are no higher than certain threshold values. Since in each settlement there is already a certain amount of pollutants, in deciding on the location of a new plant it is necessary to take into account the fact that the increase in the level of pollution must be no higher than $C_{\text{m.p}} - C_0$. The problem on determination of the territory where this condition is fulfilled can also be investigated by solving the adjoint equations. However, to estimate the aerosol concentration in a settlement depending on the site of the emission source, the function $P(r)$ in the functional F should be replaced by the function $G(r)$, which is equal to unity in the settlement and to zero in all the remaining regions. The modified functional determines within a multiplier the dependence of the aerosol concentration in the selected settlement on the location of the emission source. In the case where there are several settlements in the region, modified functionals should be calculated for each settlement. In Fig. 3, the limitations imposed by the sanitary norms on the selection of the location of an emission source (hatched region in Fig. 3a) are shown by the example of three settlements with different initial levels of pollution. It should be noted that the sanitary norms can be observed in the region where the receptor-sensitivity functional does not take the lowest values. The hazard to human health will be minimum when an emission source is positioned at a point at which the receptor-sensitivity functional is minimum for

TABLE 2. Values of Coefficients in the Formulas of the Dependence of the Standard Deviation on the Distance

Model		Unstable		Neutral		Stable	
		I	II	I	II	I	II
$\sigma_y = qx^{\nu}$							
McElroy, town [10]	VU	1.46	0.71	1.36	0.67	0.79	0.70
	UU	1.52	0.69				
ASME [11]	VU	0.359	0.93	0.346	0.77	0.304	0.71
	UU	0.328	0.87				
VOGT [12], height of ejection 50 m	A	0.87	0.81	D 0.62	0.77	E 1.69	0.62
	B	0.87	0.81			F 5.38	0.58
	C	0.72	0.78				
VOGT [12], height of ejection 100 m	A	0.23	1.0	D 0.22	0.91	E 1.69	0.62
	B	0.23	0.97			F 5.38	0.58
	C	0.22	0.94				
$\sigma_z = sx^{\gamma}$							
McElroy, town [10]	VU	0.01	1.54	0.09	0.95	0.40	0.67
	UU	0.04	1.17				
ASME [11]	VU	0.50	0.89	0.23	0.27	0.06	0.71
	UU	0.287	0.88				
VOGT [12], height of ejection 50 m	A	0.22	0.97	D 0.20	0.94	E 0.16	0.81
	B	0.22	0.97			F 0.40	0.62
	C	0.21	0.94				
VOGT [12], height of ejection 100 m	A	0.10	1.16	D 0.40	0.76	E 0.16	0.81
	B	0.16	1.02			F 0.40	0.62
	C	0.25	0.89				

Note: VU denotes high instability, UU denotes instability, and A, B, C, D, E, and F denote atmospheric-stability classes [13]. For σ_y: I) q and II) ν; for σ_z: I) s and II) γ.

the region found. In Fig. 3a, three regions where an emission source cannot be positioned because of their exceeding the maximum permissible concentrations in settlements A, B, and C are shown schematically.

It is significant that the receptor-sensitivity functional is independent of the location of the energy sources already existing in the region. Its change over the territory is determined by the wind rose, the lay of the ground, and the features of the population-density distribution.

Semiempirical Models for Calculating the Receptor-Sensitivity Functional. Semiempirical models with a parametric dependence of the standard deviations of the Gaussian distribution on x, y, and z can be used for calculating the receptor-sensitivity functional. For example, for $\mathbf{U} = (U, 0, 0)$ and a continuously working source with an intensity Q, positioned at the point $r = r_0$, the concentration distribution can be described as

$$C(x, y, z, x_0, y_0, z_0) = \frac{Q\Theta(x - x_0)}{2\pi U \sigma_x(x - x_0)\,\sigma_y(x - x_0)} \exp\left(-\frac{(y - y_0)^2}{2(\sigma_y(x - x_0))^2}\right) \times$$

$$\times \left[\exp\left(-\frac{(z - z_0)^2}{2(\sigma_z(x - x_0))^2}\right) - \exp\left(-\frac{(z - z_0)^2}{2(\sigma_z(x - x_0))^2}\right)\right],$$

where the following empirical parametric dependences on the distance are usually used for the standard deviations σ_x, σ_y, and σ_z:

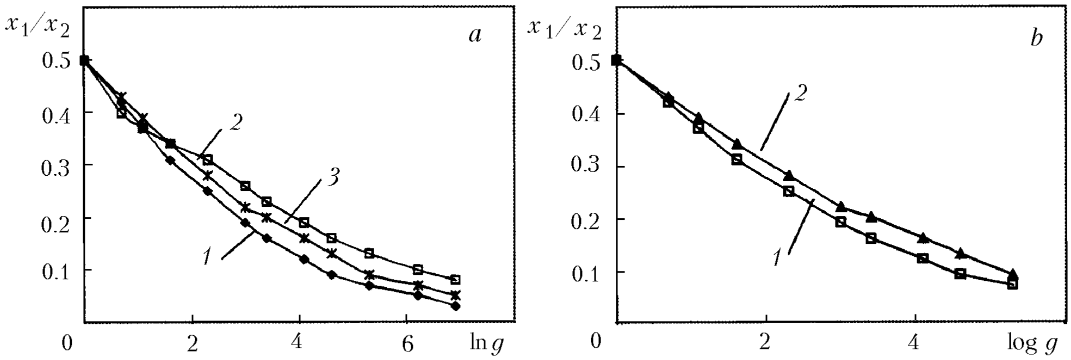


Fig. 4. Dependences of the ratio of the distances between the settlements x_1/x_2 on the parameter g, calculated by the ASME model (a) [1) neutral atmosphere; 2) unstable atmosphere; 3) stable atmosphere] and by the VOGT model (b) [1) h = 0; 2) 50 m].

$$\sigma_x(x) = \sigma_y(x) = qx^{\nu}, \quad \sigma_z(x) = sx^{\gamma}. \tag{12}$$

The values of the coefficients q, s, γ, and ν for different atmospheric states are presented in Table 2. In the problem with two settlements, the semiempirical function of receptor sensitivity has the following form:

$$F(x_0, y_0, z_0) = \frac{aP_1Q\Theta(x_0)}{\pi U_1 \sigma_x(x_0)_x \sigma_y(x_0)} \frac{\Delta t_1}{T} \exp\left(-\frac{y_0^2}{2(\sigma_y(x_0))^2} - \frac{z_0^2}{2(\sigma_z(x_0))^2}\right) +$$

$$+ \frac{aP_2Q\Theta(L - x_0)}{\pi U_2 \sigma_x(L - x_0)\sigma_y(L - x_0)} \frac{\Delta t_2}{T} \exp\left(-\frac{y_0^2}{2(\sigma_y(L - x_0))^2} - \frac{z_0^2}{2(\sigma_z(L - x_0))^2}\right). \tag{13}$$

In the case where an emission source acting near the surface must be positioned on the straight line connecting settlements A and B, the minimum value of the functional F is attained at the point

$$\frac{x_1}{x_2} = \left(\frac{P_1U_1\Delta t_1}{P_2U_2\Delta t_2}\right)^{1/(\nu+\gamma-1)}. \tag{14}$$

The distributions of the receptor-sensitivity functional for the three settlements are presented in Figs. 2 and 3. The dependences of the ratio x_1/x_2 on the parameter $g = P_1U_1\Delta t_1/(P_2U_2\Delta t_2)$ for different values of the effective height of the emission source and different atmospheric states are shown in Fig. 4.

An example of calculation of the receptor-sensitivity functional for the territory of Minsk with the use of the "Nostradamus" code is shown in Fig. 5. In the calculations, we used data on the wind rose, averaged over the period of observations of many years [14]. As expected, different regions of Minsk are not equivalent from the viewpoint of sensitivity to the location of emission sources. Noteworthy is the fact that the values of the sensitivity potential differ by more than an order of magnitude for different districts of the city. This means that in the case of existence of two identical emission sources, the hazard to the health of people living in the town can differ by more than an order of magnitude, depending on the location of the source. For a near-surface source the value of the receptor-sensitivity functional at a given point of the territory of Minsk significantly depends on the number of people living in the region of dimension σ_y^{-1}, where the aerosol concentration decreases due to convective diffusion. With increase in the height of the emission source, the contribution of more distant territories to the functional increases markedly. In this case, the asymmetry of the directions and force of the winds substantially influences its distribution.

The distribution of the functional over the territory of one district of the Minsk Region is presented in Fig. 6. The values of the receptor-sensitivity functional are higher for the characteristic placement of settlements along the

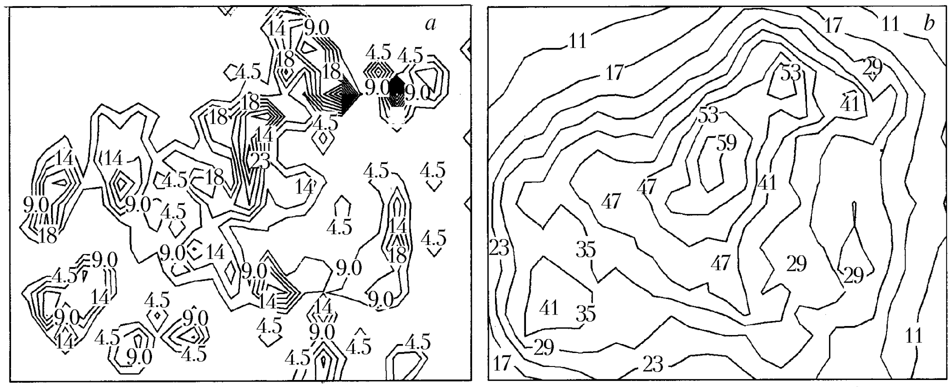


Fig. 5. Distribution of the population density (a) (thousands of people/km^2) and the functional of receptor sensitivity (b) (rel. units) over the territory of Minsk. The computational region measures 21 × 16 km, h = 0.

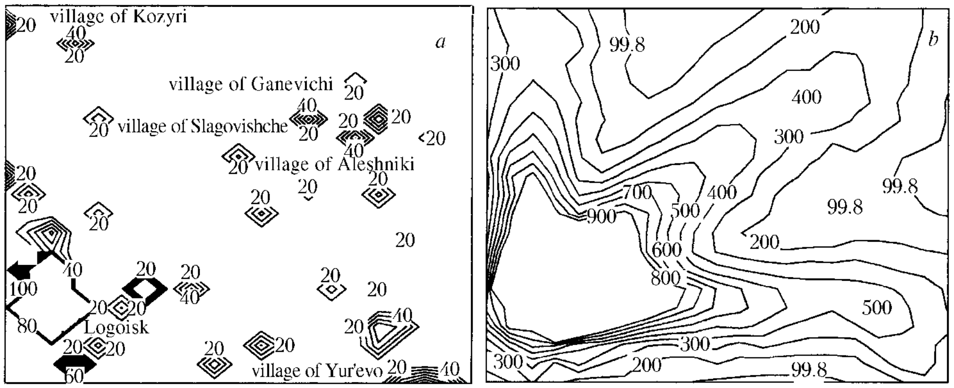


Fig. 6. Distribution of the population density (a) (people/km^2) and the functional of receptor sensitivity (b) (rel. units) over the territory of the northwest part of the Logoisk District of the Minsk Region. The computational region measures 20 × 20 km, h = 50 km.

roads in the directions selected. From the viewpoint of the hazard to human health, such a disposition of roads causes the greatest damage. The mapping of the territory by the value of the receptor-sensitivity functional makes it possible to design new detours in such a way that the hazard to human health is minimum. It is significant that any variants of the disposition of roads can quantitatively be analyzed by the value of the hazard of them to the health of people and the environment. Just as for Minsk, the receptor-sensitivity functional (see Fig. 6) is distributed irregularly, and its maximum and minimum values differ considerably.

CONCLUSIONS

1. The functional of receptor sensitivity of a territory to the location of an emission source can efficiently be calculated with the use of the mathematical apparatus of adjoint equations.

2. For the purpose of decreasing the hazard to human health and the environment, it seems reasonable to come up with a program of placement of industrial plants ejecting harmful aerosols and gases into the atmosphere for each region. Maps reflecting the distribution of the receptor sensitivity over the territory of each region can be prepared with regard for the climatic conditions, the wind rose, and the features of the lay of the ground. This should be done first when construction of objects in developing economic regions is planned; in this case, decisions reasonable from the viewpoint of protection of the environment and man could be made.

3. The mapping of the territory of a region by the value of the receptor sensitivity would also be useful in planning of measures aimed at decreasing the emission of industrial plants and transport. The distribution of the recep-

tor-sensitivity functional over the region would make it possible to evaluate the efficiency of the measures proposed from the viewpoint of decreasing the hazard to human health.

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NOTATION

α, coefficient characterizing the interaction of impurities with the underlying surface; b, coefficient of the dose–response relation; A, B, and C, settlements; C, impurity concentration in the atmosphere, kg/m^3; C^*, collective exposure to ejections of an emission source of unit intensity, sec·persons/m^3; $C_{\text{m.p}}$, maximum permissible concentration, kg/m^3; C_0, initial level of pollution, kg/m^3; F, functional of receptor sensitivity; $G(r)$, modified functional; K_x, K_y, and K_z, coefficients of diffusion in the direction of the x, y, and z axes, m^2/sec; L, distance between settlements A and B; n, number of stationary solutions; $P(r)$, population density; P_i, number of people in the ith settlement; q and ν, coefficients in the dependence of the standard deviation σ_y on the distance; $\mathbf{r}$, radius vector of a point of the computational region; $\mathbf{r}_i$, radii vectors of the sources; s and γ, coefficients in the dependence of the standard deviation σ_z on the distance; t, time, sec; T, time period of the function C, sec; $\mathbf{U}$, wind velocity, m/sec; U_i, wind velocity in stable regimes of flow, m/sec; x, y, z, coordinates of a point of the computational region; x_0, y_0, z_0, coordinates of the source; Δt_i, time of the stable regime of air masses; Σ, side cylindrical surface; Σ_0, cross section of the cylindrical surface at the level $z = 0$; Σ_H, cross section of the cylindrical surface at the level $z = H$; Q, intensity of the emission source; δ, Dirac delta function; Θ, Heaviside function; λ, constant determining the decomposition of the impurity; σ_x, σ_y, and σ_z, standard deviations; h, effective height of the emission source, m; φ, angle at which the wind velocity is directed to the x axis. Subscripts: i, number of the stationary solution, $i = 1 \ldots n$; m.p, maximum permissible; 0, initial.

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